Advanced Math

9a-2

Arithmetic Sequences

Arithmetic Sequence - All the terms have a common difference, d.

Determine whether or not the sequence is arithmetic. (pg 726)

3) 1, 2, 4, 8, 16, . . .

1 2

No!

9) 5.3, 5.7, 6.1, 6.5, 6.9, . . .

.4 .4 .4 .4

Yes.

*n*th term of an Arithmetic Sequence: $a_n = a_1 + d(n-1)$

Write the first five terms of the sequence. Determine whether the sequence is arithmetic, and if it is, find the common difference.

11) $a_n = 5 + 3n$

$8+3(n-1)$

$a_1 = 8$, $d = 3$

$a_2 = 11$

$a_3 = 14$

$a_4 = 17$

$a_5 = 20$

This is a traditionally defined sequence function.

19) $a_1 = 15$, $a_{k+1} = a_k + 4$

seed

$a_2 = 19$

$a_3 = 23$

$a_4 = 27$

$a_5 = 31$

$d = 4$

Recall: This is a recursively defined sequence.

Write the first five terms of the arithmetic sequence.

31) $a_8 = 26$, $a_{12} = 42$ (see right)

First, adjust like so:

pretend → $a_1 = 26$ (set as 1st term) $a_5 = 42$ (n) (adjust)

$8 - ? = 1$

7

$12 - 7 = n$

$5 = n$

$a_n = a_1 + d(n-1)$

$42 = 26 + d(5-1)$

$16 = 4d$

$4 = d$

Then find the true a_1:

$a_n = a_1 + 4(n-1)$

$26 = a_1 + 4(8-1)$

$26 = a_1 + 28$

$-2 = a_1$

Finish with the true equation.

$a_n = -2 + 4(n-1)$

-2, 2, 6, 10, 14 (+4, +4, +4, +4)

Find a formula for a_n for the arithmetic sequence.

43) $a_3 = 94$, $a_6 = 85$

pretend $a_1 = 94$, $a_4 = 85$

$85 = 94 + d(4-1)$

$-9 = 3d$

$-3 = d$ (1st adjust.)

$a_n = a_1 - 3(n-1)$

$94 = a_1 - 3(3-1)$

$94 = a_1 - 6$

$100 = a_1$

Then find the actual $a_n = 100 - 3(n-1)$

Sum of an arithmetic sequence: $S_n = \frac{n}{2}(a_1 + a_n)$

This is the history of how this formula came about. Notice the underlined terms add to 101. Hence, $a_1 + a_n$. Since we use two at a time, there are 50 pairs, or $n/2$. They multiply to 5050.

$1 + 2 + 3 \quad \ldots \quad + 98 + 99 + 100$

Find the sum of the first n terms of the arithmetic sequence.

55) 8, 20, 32, 44, . . . , $n = 10$

a_1 a_2

$a_n = a_1 + d(n-1)$

$a_{10} = 8 + 12(10-1)$

$a_{10} = 8 + 12(9) = 116$

$S_{10} = \frac{10}{2}(8 + 116)$

$= 5(124)$

$= 620$

Evaluate -

65) $\sum_{n=1}^{100} 5n$ $\Rightarrow a_1 = 5, a_2 = 10, a_3 = 15, \ldots, a_{100} = 500$

$S_{100} = \frac{100}{2}(5 + 500)$

$= 50(505)$

$= \boxed{25250}$

In this problem, a_{100} was easy to find. If not easy, use the a_n formula.

Assignment:
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2-24 every 4th,
26-44 even,
56-78 even,
82.